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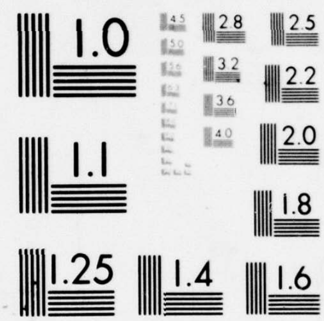

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A Study of the Error of Discretization in the Air Force Global Weather Central Boundary Layer Model

CHIEN-HSIUNG YANG

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AIR FORCE SYSTEMS COMMAND, USAF

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FOR THE COMMANDER

John Howard

Chief Scientist

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The computational structure in the vertical of the Air Force Global Weather Central Boundary Layer Model is analyzed. The approach used is to measure the degree of inaccuracy in the numerical solution of a time-dependent problem by the error of the steady-state solution incurred through discretization procedures. It is found that the present structure contains defects that are amenable to improvements that can be implemented without a substantial increase in computational effort.

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A Study of the Error of Discretization in the Air Force Global Weather Central Boundary Layer Model

1. INTRODUCTION

This report summarizes the results of an investigation that was undertaken to examine how the method of discretization and the structure of the computational grid in the vertical direction employed in the Air Force Global Weather Central Boundary Layer Model (AFGWC-BLM) might influence the forecast accuracy of the model.

AFGWC-BLM simulates the evolution of the planetary boundary layer over a continental area through the numerical solution of an initial-boundary value problem posed on a set of grid points distributed within the domain of interest. A measure of forecast accuracy may, therefore, be obtained from the aggregate of the differences between the predicted and observed values of the variable of interest. The effects that a particular part or substructure of the model have upon forecast performance can be studied by regarding the whole model as a system and by carrying out numerical experiments in which the part under investigation is kept under control. Such a system approach for the study of prediction models was proposed as a standard procedure to be used for diagnosis and improvement of dynamical prediction models in a previous report.[1] In the present study, a different approach is used for the following reasons: AFGWC-BLM is

(Received for publication 15 April 1977)

1. Yang, Chien-hsiung (1976) A Proposed Procedure for Diagnosis and Improvement of Dynamical Prediction Models, AFGL-TR-75-0079.

constructed with the aid of many modeling assumptions and many mathematical approximations. This implies that the model is endowed with as many independent sources of error as the number of such artifices. To make matters worse, there is virtually no way of ascertaining how well these artifices represent the real state of the atmosphere at any given specific time and place. It is, therefore, extremely difficult to accurately trace the forecast error of the model to a particular source or cause without extensive experimentation.

However, it is possible to make a reasonable assessment of the influence or significance that a particular substructure of the model has upon forecast accuracy, if one does not insist upon an answer that establishes a cause-and-effect relationship. This is done by isolating the substructure of interest from the model and by examining its characteristics independent of the rest of the model. This method of investigation may be applied if evaluation does not require observational data of the real atmosphere, and is, therefore, particularly suitable for studying the nature of the mathematical structure of the model. The subject of this investigation is one example of such a method. However, as will be shown in Section 2, even this method requires considerable simplification and qualification in order to obtain definite conclusions.

2. SCOPE AND METHOD OF INVESTIGATION

To explain the rationale underlying the investigation, let us start by considering a typical prediction problem: Given a suitable set of initial and boundary conditions, we search for the solution $y(\vec{x}, t)$ in a bounded continuous domain D, $\vec{x} \subset R$, $0 < t \leq T$, that satisfies differential equation

$$\frac{\partial y}{\partial t} = f(\vec{x}, y, t) \qquad \begin{matrix} x \subset R \\ 0 < t \leq T \end{matrix} , \tag{1}$$

in which f is generally a nonlinear function of the variables concerned. It is assumed that the existence and uniqueness of the solution is beyond doubt and, therefore, presently out of question. We shall denote by y_D the solution satisfying Eq. (1) and the other accompanying conditions.

The present state of the art in dealing with this problem, except for a very few cases of extreme simplicity and limited practical value, usually requires two steps: discretization and linearization. The continuous domain D is replaced by a finite set Δ of grid points $(\vec{x}_i, t_n)$ formed by nets covering D. The value $y(\vec{x}_i, t_n)$ at each of these grid points is sought. Function f is linearized so that the discrete analog of the differential equation is reduced into a system of algebraic equations of the form

$$y_\Delta(\vec{x}_i, t_{n+1}) = y_\Delta(\vec{x}_i, t_n) + F(y_\Delta(\vec{x}_k, t_n), \vec{x}_k, t_n) . \tag{2}$$

The set $\{y_\Delta(x_i, t_{n+m}), x_i \subset R, 1 \leq m \leq N\}$ constitutes the prediction, and will be referred to as the discrete solution.

A direct measure of the accuracy of such a prediction can be readily obtained from the differences $y_\Delta(\vec{x}_i, t_{n+m}) - y_D(\vec{x}_i, t_{n+m})$, except for the fact that there is generally no analytic solution whose values can be obtained as readily as those of $y_\Delta(\vec{x}_i, t_{n+m})$ for any practical function $f(y, x, t)$. A substitute is therefore proposed that, while not directly representative of prediction accuracy, expresses the degree of inaccuracy incurred through the process of discretization and can be used for assessing the merit of any particular procedure in comparison with other procedures performing the same function. The substitute is the difference between two steady-state solutions (y_S and y_Σ, respectively) of Eqs. (1) and (2) on the set of grid points, where y_S and y_Σ are obtained by removing the time dependence from both equations. The quantity measures one aspect of inaccuracy, even for the time-dependent problem, since it induces a change with time in the discrete system, even when the continuous system is in the stationary state. We shall call it the discretization error in the steady-state solution.

AFGWC-BLM employs a computational grid that has a net of uniform spacing on a stereographic projection in the horizontal direction and a constant time-step for the entire period of time integration. In the vertical, however, the grid is made up of eight unevenly distributed levels. They are the surface level, to which the height of 1.25 m above ground level (AGL) is assigned in the computation, 50 m, 150 m, 300 m, 600 m, 900 m, 1200 m, and 1600 m AGL.

The model simulates the vertical turbulent transfer of heat and moisture with the use of eddy diffusivity. The eddy diffusivity is defined to be dependent on the velocity and temperature distributions in the vertical and, therefore, varies with both time and height.

Since all spatial derivatives in the continuum are replaced by finite differences on the computational grid, our attention was drawn particularly to the manner in which discretization is carried out in the vertical. A simple model that incorporates all aspects of concern is obtained in the Dirichlet problem of heat conduction defined below to find $y(z)$ that satisfies (A) and (B):

$$\left.\begin{array}{ll}
\text{(A) Differential Equation} & \\
\quad (K(z)y')' = S(z) & \\
 & 1 > z > 0 \\
\quad K(z) \geq K_0 > 0 & \\
\text{with } y' = dy/dz & \\
\text{(B) Boundary Conditions} & y(0) = y_0 \text{ and } y(1) = y_1 .
\end{array}\right\} \tag{3}$$

Here, two parameter functions K(z) and S(z) are so prescribed that the existence of a continuous solution is insured. It is readily shown that solution $y_S(z)$ may be given by

$$y_S(z) = y_o + \int_0^z \frac{C_1 + \int_0^u S(v)dv}{K(u)} du ,$$

where

$$C_1 = \frac{(y_1 - y_0) - \int_0^1 \left(\int_0^z S(u)du/K(z) \right) dz}{\int_0^1 dz/K(z)} . \tag{4}$$

The solution can be easily evaluated when K(z) and S(z) are simple functions of z.

Next, the discrete version of the Dirichlet problem (3) is obtained by using a given method of discretization. The discrete version takes the form of a system of equations whose solution is $y_\Sigma(z_i)$, which together with $y_S(z_i)$ defines the discretization error in the steady-state solution. Comparisons of discretization errors among a class of comparable methods of discretization will then enable us to assess the relative merits of individual methods.

Because of the limited significance of the discretization error in the steady-state solution, we did not believe it worthwhile to generalize the parameter functions or the boundary conditions. If the results on simple parameter functions afford an unequivocal evaluation, we feel that they should be sufficient for us to make choices. The investigation is thus confined to cases that arise from combining one eddy diffusivity and one source function from the sets listed below. Profiles of some of the eddy diffusivities and the source functions are shown in Figures 1 and 2 respectively. Those that are not shown may be inferred easily from those shown.

(A) Eddy Diffusivity, K(z)

$$K_1 = -49(z - .25)^2 + 28.5625$$
$$K_2 = -49(z - .375)^2 + 20.140625$$
$$K_3 = -49(z - .50)^2 + 13.25$$
$$K_4 = 100(z - .32)^2 + 1.00$$
$$K_5 = 100(z - .32)^2 + 0.10$$
$$K_6 = 100(z - .32)^2 + 0.01$$

(B) Source Function, S(z)

$$S_1 = 0$$
$$S_2 = 10(1 - z^2)$$
$$S_3 = -40(z - 1/2)^2 + 10$$
$$S_4 = 20 - S_3$$
$$S_5 = 10z(2 - z)$$
$$S_6 = -S_3$$

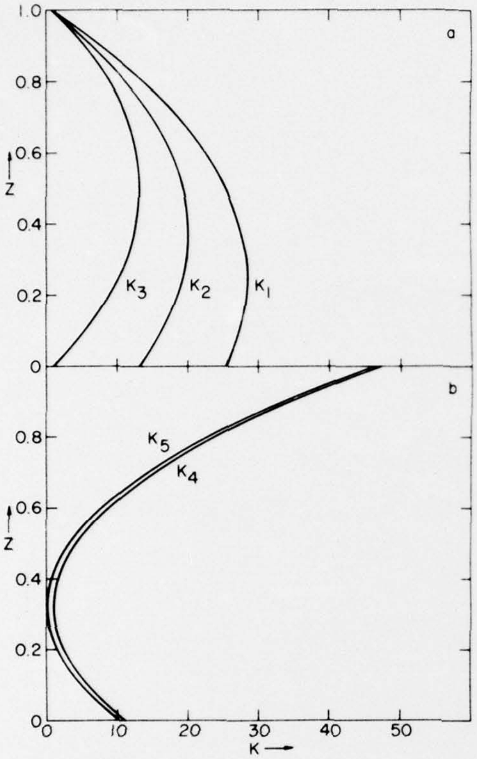

Figure 1. Profiles of Some of the Eddy Diffusivities Used in the Study

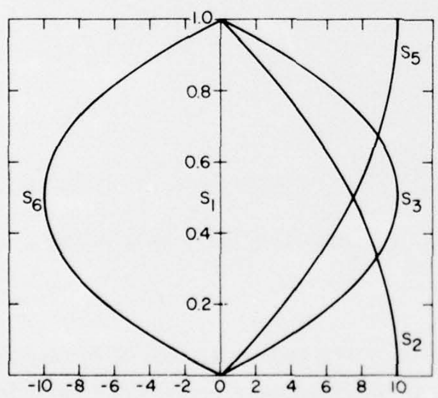

Figure 2. Profiles of Some of the Source Functions Used in the Study

(C) Boundary Conditions

$$y_0 = 0, \; y_1 = 1.0 \quad .$$

The profiles of some continuous solutions are shown in Figure 3, where each profile is identified by a pair of parameters as (K_i, S_j).

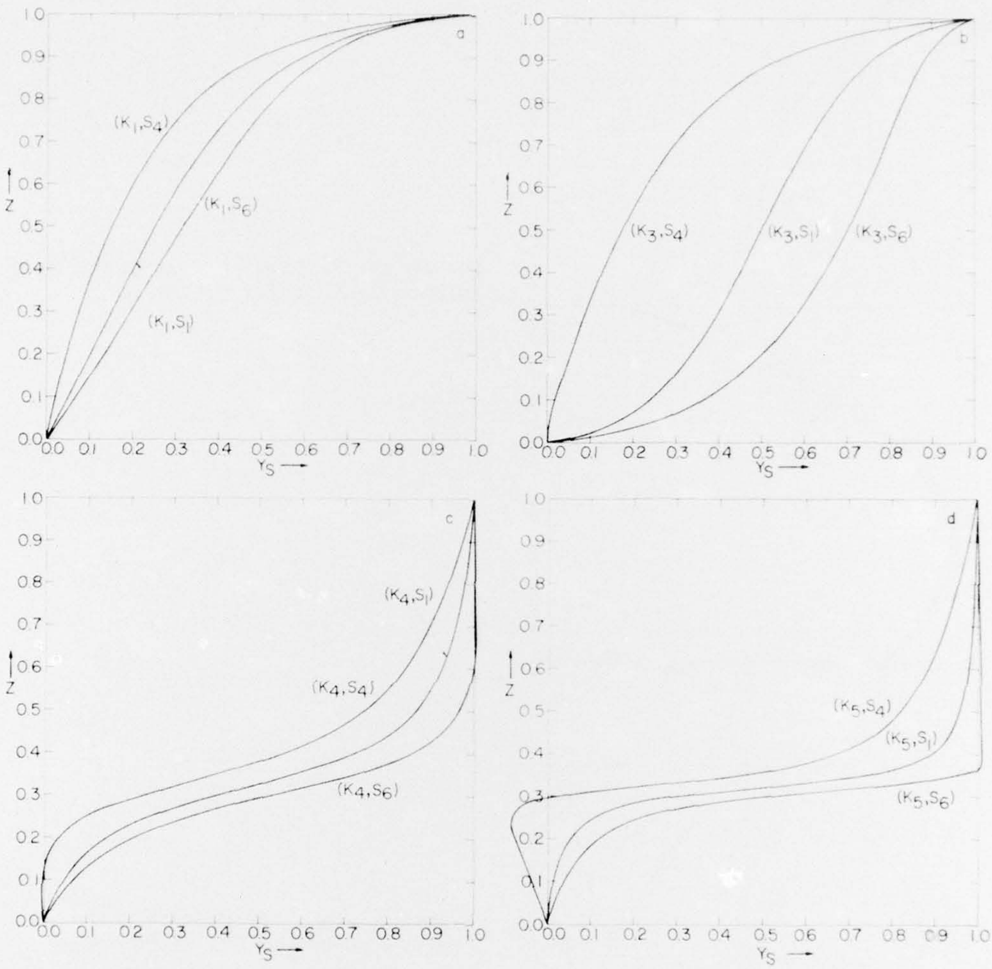

Figure 3. Sample Continuous Functions, y_S, Identified by a Pair of the Parameter Functions (K_i, S_j)

Two different finite-difference formulations of the left-hand side of the equation of heat conduction, Eq. (3), and three different computational grids were considered. Comparisons were made to put a separate focus on each of two different aspects of discretization; namely, the finite-difference formulation and the structure of computational grids represented by density and distribution. Each subject will be treated separately in Section 3.

3. RESULTS OF INVESTIGATION

3.1 Finite-Difference Formulations

We consider the following two finite-difference formulations for the expressions $(K(z)y')'$ on a computational grid $(z_i, \; i=1,2,\ldots,N)$. Type 1 is suggested by writing $(K(z)y')' = K(z)(y')' + K'y'$. Thus, Eq. (3) becomes discretized as

$$\frac{2K_i}{(z_{i+1} - z_{i-1})}\left[\frac{y_{i+1} - y_i}{z_{i+1} - z_i} - \frac{y_i - y_{i-1}}{z_i - z_{i-1}}\right]$$

$$+ \frac{K_{i+1} - K_{i-1}}{z_{i+1} - z_{i-1}} \cdot \frac{y_{i+1} - y_{i-1}}{z_{i+1} - z_{i-1}} \equiv \Delta_i^{(1)} = S_i \qquad (5)$$

$$i = 2, \ldots, (N-1) \; ,$$

where $K_i = K(z_i)$, $y_i = y(z_i)$, $z_1 = 0$, and $z_N = 1$.

This is the version employed in AFGWC-BLM. It is based on the notion that a differentiation at a point is to be replaced by the mean rate of change of the quantity over the interval between the two points, one on each side, next to the point of concern. It is readily shown that this formulation is consistent and that the local truncation error is given by

$$\epsilon_{1i} = \Delta_i^1 - (K(z)y')'_i$$

$$= \left[\frac{1}{3}K_i(y''')_i + \frac{1}{2}(K')_i(y'')_i + \frac{1}{2}(K'')_i(y')_i\right](zi_{+1} - 2z_i + z_{i-1})$$

$$+ O(\Delta z^2) \; . \qquad (6)$$

Type 2 is obtained by approximating the value of K(z) at the midpoint of an interval by the average of its values on the two boundaries of the interval. It is given by

$$\frac{2}{(z_{i+1} - z_{i-1})}\left[\frac{K_i + K_{i+1}}{2} \cdot \frac{y_{i+1} - y_i}{z_{i+1} - z_1} - \frac{K_{i-1} + K_i}{2} \cdot \frac{y_i - y_{i-1}}{z_i - z_{i-1}}\right]$$

$$\equiv \Delta_i^{(2)} = S_i \qquad (7)$$

$$i = 2, \ldots, (N-1) \; .$$

It may also be shown that the Type 2 formulation is consistent and has the same truncation error up to the first order as does Type 1; that is,

$$\epsilon_{zi} = \Delta_i^{(2)} - (K(z)y')'_i = \Sigma_{1i} + O(\Delta z^2) . \tag{8}$$

Notwithstanding these similarities, there is a subtle difference between the two formulations. The difference is revealed in the characteristics of the resultant coefficient matrices that become pertinent only when the steady-state solution is sought. It becomes more obvious when Eqs. (5) and (7) are rewritten, respectively, as

$$\begin{aligned}
&\left[\frac{2K_i}{(z_i - z_{i-1})} - \frac{(K_{i+1} - K_{i-1})}{(z_{i+1} - z_{i-1})}\right] y_{i-1} \\
&\quad - \left[\frac{2K_i}{(z_{i+1} - z_i)} + \frac{2K_i}{(z_i - z_{i-1})}\right] y_i \\
&\quad + \left[\frac{2K_i}{(z_{i+1} - z_i)} + \frac{(K_{i+1} - K_{i-1})}{(z_{i+1} - z_{i-1})}\right] y_{i+1} + (z_{i+1} - z_{i-1}) S_i
\end{aligned} \tag{5'}$$

and

$$\begin{aligned}
&\frac{(K_i + K_{i-1})}{(z_i - z_{i-1})} y_{i-1} \\
&\quad - \left[\frac{(K_i + K_{i-1})}{(z_i - z_{i-1})} + \frac{(K_i + K_{i+1})}{(z_{i+1} - z_i)}\right] y_i \\
&\quad + \frac{(K_i + K_{i+1})}{(z_{i+1} - z_i} y_{i+1} = (z_{i+1} - z_{i-1}) S_i .
\end{aligned} \tag{7'}$$

$$i = 2, \ldots, (N-1) .$$

So long as $K_i > 0$, for all i's, as has been assumed in the definition of the problem, the coefficients a_{ik} of y_k in the i-th equation in Eq. (7') are seen to satisfy $a_{ii} < 0$; $a_{ik} \geqq 0$ if $i \neq k$; and $\sum_{\substack{k=1 \\ k \neq i}}^{N} |a_{ik}| = |a_{ii}|$ for all $i=2,\ldots,(N-1)$. The same, however, does not hold true for the coefficients a_{ik} of y_k in Eq. (5').

This means that when the system of equations for y_k (incorporating the boundary conditions) is obtained, the coefficient matrix formed from Eq. (7') is irreducibly diagonally dominant, irrespective of the K profile and of the computational grid involved, and is, therefore, always non-singular. The same cannot be said of the corresponding coefficient matrix formed from Eq. (5'). This, however, does not imply that the latter is necessarily singular. In fact, with the restricted structure that it has, it is rather a fortuitous and rare occurrence that the coefficient matrix associated with Eq. (5') turns out to be singular in practice. The fact that one is and the other is not diagonally dominant is obviously sufficient to produce a

significant difference in discretization error between the two formulations, as described below.

Two typical patterns of discretization error seem to emerge from all the cases considered. Figures 4a and 4b present each of these types, using the results obtained with the computational grid "10U" that subdivides the domain $1 \geq z \geq 0$ into 10 equal subintervals. In each of these figures there are three entries: the continuous solution $y_S(z)$ shown by a solid curve; the discretization error with Type 1 finite-difference formulation at each grid point represented by ⊙ ; and the discretization error with Type 2 formulation represented by x. Note that while the range and scale of y_S remain the same in both figures, those for discretization error vary from one to the other.

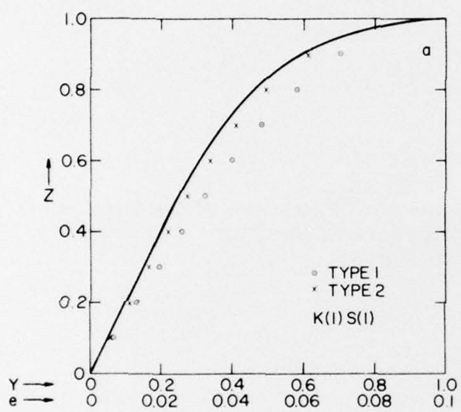

Figure 4a. Pattern A of the Discretization Errors of the Finite-Difference Formulations

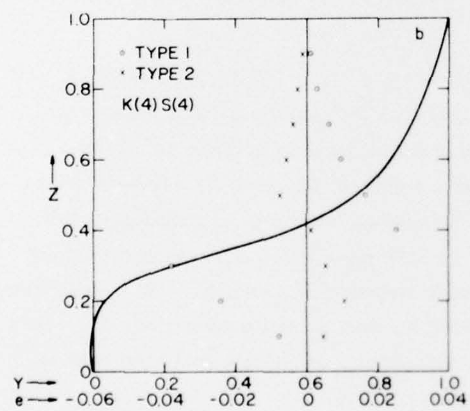

Figure 4b. Pattern B of the Discretization Errors of the Finite-Difference Formulations

Pattern A, typified by those shown in Figure 4a, is observed in eddy diffusivities K_1, K_2, and K_3 and in association with all the source functions considered. The discrete solutions of both Type 1 and 2 formulations are biased toward the

straight line joining the two boundary values y_0 and y_1 and, therefore, have profiles of great similarity. The magnitudes of the discretization errors increase with the local curvature of the continuous solution y_S. Differences among the eddy diffusivities and among the source functions produce differences in the magnitude of individual errors, but do not appear to create any outstanding change either in the distributions or in the relation between the two types.

In all cases computed, the error in Type 2 was smaller than that in Type 1 at a ratio that remained nearly constant at all grid points. Another example, which is associated with a slightly different profile of continuous solution y_S, is shown in Figure 5a to illustrate the common features of Pattern A.

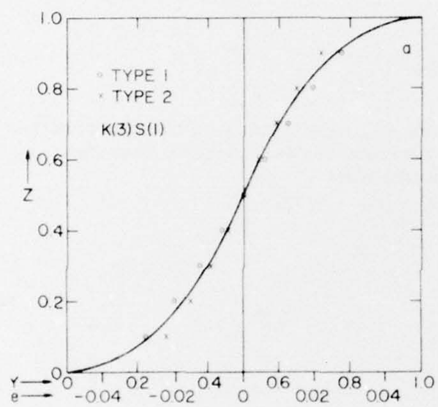

Figure 5a. Pattern A of the Discretization Errors of the Finite-Difference Formulations

Pattern B, characterized by the distributions shown in Figure 4b, is observed in eddy diffusivities K_4, K_5, and K_6. One essential difference of this type from Type A, readily noticeable in Figure 4b, is the contrast between the two types of formulations. This contrast becomes starker as eddy diffusivity is changed from K_4 to K_5, as shown in Figure 5b. The only difference between K_4 and K_5 is the minimum value of eddy diffusivity (min $K_4 = 1$, min $K_5 = 0.1$). As shown earlier in Figure 1b, there is little difference in profile between K_4 and K_5. K_6, which has the minimum of 0.01, is indistinguishable from K_5 on the scale of Figure 1b. However, when K_6 is used, the Type 1 discretization error becomes so large that it cannot be put in Figure 5b.

In Type B, the discretization error of Type 2 formulation remains well-behaved, while that of Type 1 shows a distinct mark of pathological disorder. Table 1 illustrates this by presenting the errors of the largest magnitude for both formulations under a variety of conditions. To be sure, the discretization errors in both

formulations become larger as the approximate representation afforded by the discrete system becomes worse. It is, however, clear that Type 1 formulation is definitely inferior to Type 2.

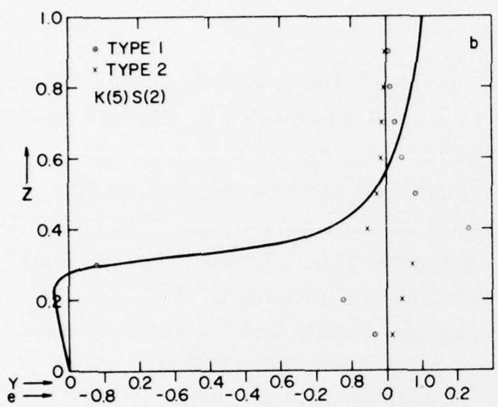

Figure 5b. Pattern B of the Discretization Errors of the Finite-Difference Formulations

Table 1. Errors of Largest Magnitude ($\times 10^{-4}$) in Pattern B

Source Function	Eddy Diffusivity					
	K_4		K_5		K_6	
	Type 1	Type 2	Type 1	Type 2	Type 1	Type 2
S_1	347	-116	-5863	-721	-44443	1824
S_2	273	-89	-8190	560	-60903	2447
S_3	-319	-85	-7624	671	-56635	2263
S_4	-377	108	-8788	829	-65163	2627
S_5	273	-90	-6760	554	-50535	2041
S_6	449	-167	-4102	-950	-32251	1385

A close examination reveals that the pattern exhibited by the discretization error of Type 1 formulation is closely associated with either weak or the lack of diagonal dominance in the coefficient matrix of the system of equations that determines the discrete solution. For example, even though K_4 provides a diagonally dominant coefficient matrix, the degree of dominance at grid points near the level of the minimum diffusivity is less than that of those in the vicinity of the maximum

diffusivity in K_1, K_2, or K_3. In K_5 and K_6 diagonal dominance does not exist at all at the two grid points (levels 0.3 and 0.4) adjacent to the level of minimum diffusivity (z=0.32), and the disparity is greater in K_6 than in K_5. This is the reason why the disorder is observed in the discretization error of the Type 1 formulation and not in that of Type 2, which preserves diagonal dominance under all conditions by nature of its construction. We may also note in Table 1 that the error of the largest magnitude for the Type 2 formulation is in every case smaller than that for Type 1.

3.2 Structure of Computational Grids

Given the domain and range of prediction, density of the computational grid for a dynamical prediction model is restricted within a well-defined range that is largely determined by such factors as size and speed of the computer employed and the preparation time available for the required forecast. With the understanding that AFGWC-BLM was designed to make maximum use of the computer capability, we compared three computational grids for accuracy using the Type 2 finite-difference formulation. They are: "10U", the grid that subdivides the domain, $1 \geq z \geq 0$, into 10 equal subintervals; "7U", the grid that subdivides the domain into 7 equal subintervals; and "7N", the grid consisting of levels z = 1/30, 1/10, 1/5, 2/5, 3/5, 4/5, and 1. The last grid very closely resembles the vertical grid of AFGWC-BLM, and is intended to represent the latter in relation with the other two.

Figure 6 assembles six different samples selected to represent the whole spectrum of variation within the total population considered. The discretization error at each grid point of the steady-state solution obtained with each of the three grids is shown in these samples. It is obvious that the relative merit of each of the three grids changes from one case to the next. It is, however, not too difficult, even on the basis of these six samples, to arrive at an overall judgment that ranks "10U", "7U", and "7N" in the order of decreasing accuracy.

Table 2 summarizes the mean absolute errors of the three grids for six diffusivities and three source functions. Since the errors are defined on a different set of grid points for a different grid, we calculated various statistics and tried to see if a meaningful comparison could be made with such statistics. Thus, in each eddy diffusivity in Table 2, the first line gives the nine-point average of absolute errors from "10U", the second the six-point average from "7N", and the third the six-point average from "7U". Because of the differences in composition of these averages, they are not directly comparable. However, the three values on the last line (the first two being the mean absolute errors from z = 1/5, 2/5, 3/5, and 4/5 that are common to both "10U" and "7N", and the last the average of absolute errors from z = 2/7, 3/7, 4/7, and 5/7 on "7U") appear to be quite compatible with each other, particularly when taken with cognizance of the other line in each column. The table strongly suggests that "10U" can reduce the discretization error measured by mean absolute error to half that of "7N", and even "7U" can be reduced by approximately 20 percent in the majority of cases considered.

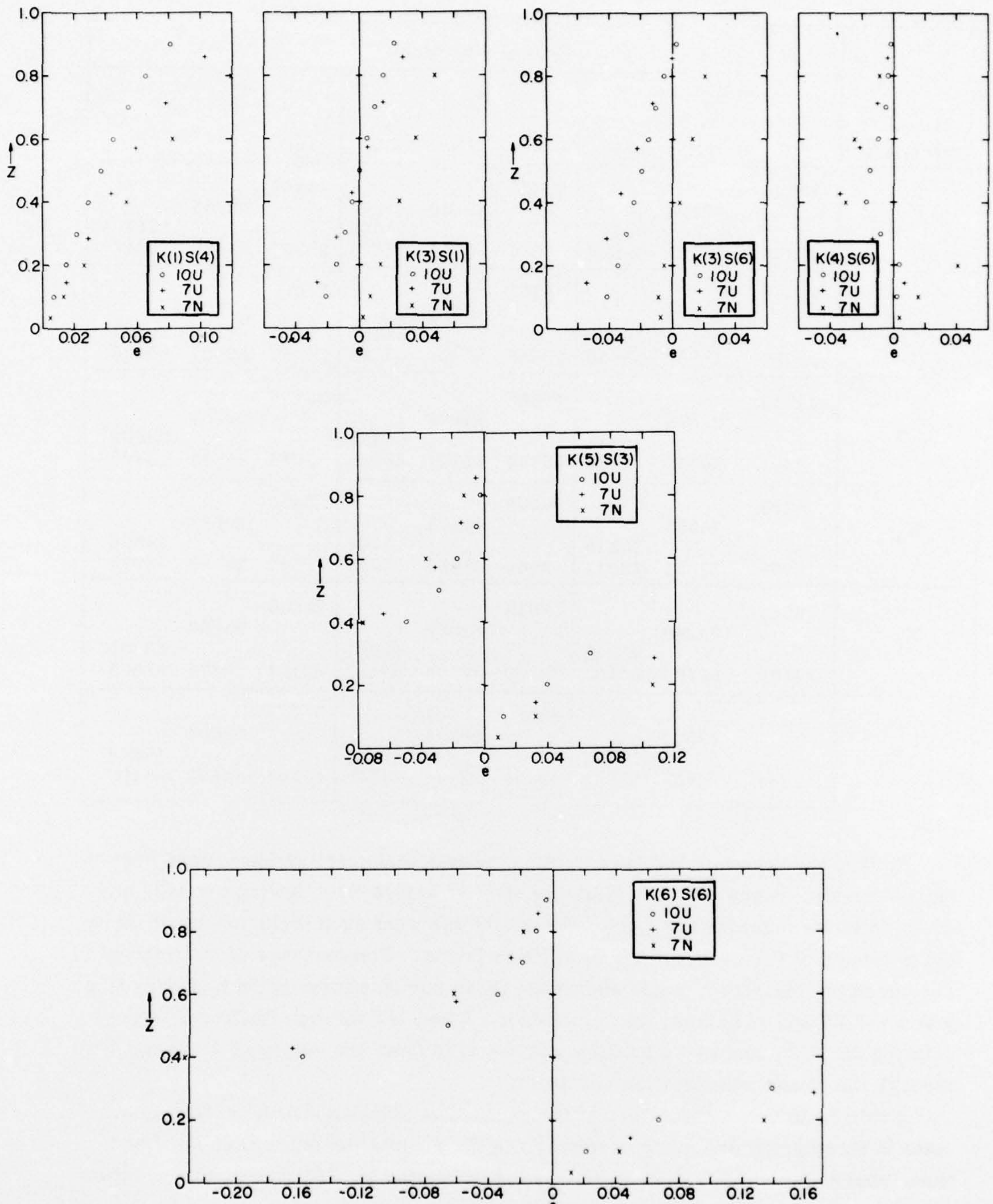

Figure 6. Samples of Discretization Errors in the Three Computational Grids Used in the Study

Table 2. Mean Absolute Discretization Errors ($\times 10^{-6}$)

Eddy Diffusivity	Source Function								
	S_1			S_4			S_6		
	10U	7N	7U	10U	7N	7U	10U	7N	7U
K_1	29829	37378	41143	39430	49730	54668	26349	42866	36652
	29121	52727	39534	38564	70144	52635	25387	61479	35347
K_2	25977	33873	36459	38271	49913	54151	21574	27568	30637
	25660	47006	35655	37856	69617	53127	21351	38691	30128
K_3	11215	21676	15413	45807	63898	65193	20118	9750	26861
	9694	30488	9615	46174	82227	65795	19661	10733	26785
K_4	5203	18503	10816	4649	15603	9138	5450	20823	14586
	6933	22084	13011	4948	17859	9643	8302	25799	18933
K_5	26640	53249	43915	22693	44927	46661	27660	53399	45200
	34395	69404	55702	28666	57104	59137	41181	80834	57495
K_6	56457	73500	77193	60660	63794	57261	57835	83804	79874
	57454	97266	101899	49729	82966	71249	67705	112428	105100

So far, our attention has been on the synopsis of discretization errors over the entire domain. Since the nonuniformity of "7N" arises from having $z = 1/30$ and $z = 1/10$ in the computational grid, one might ask what such inclusion would do to the accuracy of the local estimates at these levels. Comparisons of discretization errors were, therefore, made among the three computational grids by estimating y at $z = 1/30$ and $1/10$ from the values at $z = 0$ and $1/7$ through the linear interpolation on "7U", and by estimating y at $z = 1/30$ from the values at $z = 0$ and $1/10$ through the linear interpolation on "10U".

Table 3, in which the values of the continuous solution and the errors of estimate in three grids are given, summarizes the results for six diffusivities and three source functions. It is found that the estimates in "10U" are more accurate than those in others at both levels in the great majority of the cases. It is also true that the estimate in "7U" at $z = 1/10$ is generally better than that in "7N". On the other hand, at $z = 1/30$, "7N" and "7U" are about even in accuracy.

Table 3. Local Discretization Errors at z = 1/30 and 1/10 ($\times 10^{-6}$)

Eddy Diffusivity	Level		Source Function								
			S_1			S_4			S_6		
			10U	7N	7U	10U	7N	7U	10U	7N	7U
K_1	1/30	y		17839			6287			24089	
		e	1320	3394	1954	3138	4516	4486	1034	2849	1334
	1/10	y		52183			20655			70259	
		e	5694	9968	7195	7619	13288	11663	5110	8430	6009
K_2	1/30	y		26539			6203			37475	
		e	132	4872	-79	3893	5503	5499	-1005	2771	-1836
	1/10	y		74097			21236			104274	
		e	5917	10341	5283	9052	15507	13871	4986	7876	2642
K_3	1/30	y		128936			1797			195136	
		e	-55876	1589	-68538	11981	18743	14384	-87755	-7287	-107527
	1/10	y		240957			18820			363361	
		e	-21777	6514	-59762	22513	35737	29722	-41217	-8282	-10053
K_4	1/30	y		12164			-3045			18536	
		e	4346	4965	8331	3748	4738	7497	5598	4794	10442
	1/10	y		45955			-2865			69207	
		e	3574	17716	15529	4973	17444	16220	3194	16945	17726
K_5	1/30	y		3793			-12371			9961	
		e	5349	8760	10041	4236	8451	8330	7075	8835	12973
	1/10	y		14770			-38241			37840	
		e	12656	33115	26732	13836	32698	26118	13268	33212	30962
K_6	1/30	y		1173			-15030			7123	
		e	6801	10782	11688	5527	10406	9781	8774	11063	14913
	1/10	y		4582			-48647			26873	
		e	19340	41155	34002	20139	40496	32899	20819	42046	39234

Putting these results together, it is clear that we have not only failed to uncover any definite advantage of the nonuniform grid, but also have convincingly shown that a uniform grid is preferred to a nonuniform one, even at a density as low as those considered in the present investigation, unless there is a definite and stringent restriction on the parameter functions that makes the choice of an uneven grid indisputably more preferrable.

4. CONCLUSIONS

Using discretization error in the steady-state solution as a measure of inaccuracy for the time-dependent solution, we investigated the relative accuracy of discrete solutions obtained with a number of discretization procedures that were comparable in operational aspects with that employed in AFGWC-BLM. We have come to the following two major conclusions:

(1) The finite-difference formulation presently used in AFGWC-BLM has an undesirable feature that can be removed entirely by another formulation. The latter is of the same accuracy in local truncation error and requires about the same computational effort as the present one.

(2) The computational grid currently employed in AFGWC-BLM not only shows no distinct advantage, it also exhibits characteristics that are considered to be inferior to those of the uniformly space grid. Both types of grids require the same amount of storage space and about the same amount of computation time.